

$$e) \frac{(a^2 b^3 c^4)^3}{(a^3 b^2)^3 c^6} = \frac{a^6 b^9 c^{12}}{a^9 b^6 c^6} = \frac{b^3 c^6}{a^3}$$

$$f) (5a^3)^2 \cdot \frac{1}{5} \cdot a^4 = \frac{5^2 \cdot a^6 \cdot a^4}{5} = 5 \cdot a^{10}$$

$$g) x^{\frac{1}{2}} \cdot x^{-1.5} \cdot x = x^{0.5 - 1.5 + 1} = x^0 = 1$$

$$h) \left(\left(\left(z^{\frac{1}{2}} \right)^{\frac{3}{4}} \right)^{\frac{4}{5}} \right)^{\frac{5}{4}} = z^{\frac{1}{2} \cdot \frac{3}{4} \cdot \frac{4}{5} \cdot \frac{5}{4}} = z^{\frac{1}{2}}$$

10. a) $x^{-\frac{3}{4}} = \frac{1}{\sqrt[4]{x^3}}$

b) $\left(\frac{1}{x}\right)^{-\frac{4}{8}} = (x^{-1})^{-\frac{4}{8}} = x^{\frac{4}{8}} = x^{\frac{1}{2}} = \sqrt{x}$

c) $a^{\frac{1}{3}} b^{\frac{2}{3}} = (ab^2)^{\frac{1}{3}} = \sqrt[3]{ab^2}$

d) $\left(\frac{x}{y}\right)^{\frac{1}{2}} \left(\frac{y^2}{x}\right)^{\frac{1}{2}} = \frac{\sqrt{x}}{\sqrt{y}} \frac{y}{\sqrt{x}} = \frac{y}{\sqrt{y}} = y \cdot y^{-\frac{1}{2}} = y^{\frac{1}{2}} = \sqrt{y}$

11. a) $\sqrt{x \cdot \sqrt{x}} = (x \cdot x^{\frac{1}{2}})^{\frac{1}{2}} = x^{\frac{1}{2}} \cdot x^{\frac{1}{4}} = x^{\frac{3}{4}}$

b) $2 \cdot \sqrt{y^{-3}} \cdot x^2 \cdot \sqrt{y^5} = 2 \cdot x^2 \cdot y^{-\frac{3}{2}} \cdot y^{\frac{5}{2}} = 2 \cdot x^2 \cdot y$

c) $\frac{\sqrt[4]{x^2}}{\sqrt{y}} \cdot \frac{\sqrt[3]{y^6}}{\sqrt{x}} = \frac{x^{\frac{1}{2}} \cdot y^2}{y^{\frac{1}{2}} x^{\frac{1}{2}}} = y^{\frac{3}{2}}$

d) $\sqrt[4]{a^{-3} \cdot \sqrt[3]{a^{-3}}} = (a^{-3} \cdot (a^{-3})^{\frac{1}{3}})^{\frac{1}{4}} = (a^{-3} \cdot a^{-1})^{\frac{1}{4}} = (a^{-4})^{\frac{1}{4}} = a^{-1}$

$$(12.) \text{ a) } \frac{\sqrt[3]{(a+b)^4}}{\sqrt[3]{a+b}} = \frac{(a+b)^{\frac{4}{3}}}{(a+b)^{\frac{1}{3}}} = a+b$$

$$\text{b) } \sqrt{x+4} \cdot \sqrt{2x+8} = \sqrt{(x+4)(2x+8)} = \sqrt{2x^2 + 8x + 8x + 32}$$

$$= \sqrt{2x^2 + 16x + 32} = \sqrt{2 \cdot (x^2 + 8x + 16)} = \sqrt{2 \cdot (x+4)^2} = \sqrt{2} \cdot (x+4)$$

$$\text{c) } \sqrt{\frac{a}{4b^2}} \cdot \frac{b}{\sqrt{a}} = \frac{\sqrt{a}}{\sqrt{4b^2}} \cdot \frac{b}{\sqrt{a}} = \frac{b}{2b} = \frac{1}{2}$$

LOGARITHMUS

$$(13.) \text{ a) } \log_{10} 10000 = 4$$

$$\text{b) } \log_3 27 = 3$$

$$\text{c) } \log_{10} \left(\frac{1}{100} \right) = -2$$

$$\text{d) } \log_2 64 = 6$$

$$\text{e) } \ln(e) = 1$$

$$\text{f) } \ln(1) = 0$$

$$\text{g) } \ln(e^2) = 2$$

$$\text{h) } \ln(e^{x^2}) = x^2$$

$$(14.) \text{ a) } \ln\left(\frac{x+y}{e}\right) = \ln(x+y) - \ln(e) = \ln(x+y) - 1$$

$$\text{b) } \ln\left(\frac{y}{z}\right) + \ln\left(\frac{z}{y}\right) = \ln(y) - \ln(z) + \ln(z) - \ln(y) = 0$$

$$\text{c) } \frac{\ln(e^{x^2-y^2})}{x-y} - x = \frac{\ln\left(\frac{e^{x^2}}{e^{y^2}}\right)}{x-y} - x$$

$$= \frac{\ln(e^{x^2}) - \ln(e^{y^2})}{x-y} - x = \frac{x^2 - y^2}{x-y} - x$$

$$= \frac{(x-y)(x+y)}{x-y} - x = x+y - x = y$$

$$\textcircled{15.} \text{ a) } \ln(a) - \ln(b) + \ln(c) = \ln\left(\frac{a}{b} \cdot c\right)$$

$$\text{b) } \ln(1) + \ln(5) - \ln(a) = \ln\left(\frac{5}{a}\right)$$

$$\text{c) } \frac{1}{5} \ln(x) + \frac{1}{2} \ln(y) = \ln(x^{\frac{1}{5}}) + \ln(y^{\frac{1}{2}}) = \ln(x^{\frac{1}{5}} y^{\frac{1}{2}})$$

$$\text{d) } \frac{1}{5} \ln(x) \cdot \frac{1}{2} \ln(y) = \ln(x^{\frac{1}{5}}) \cdot \ln(y^{\frac{1}{2}})$$

$$\begin{aligned} \text{e) } \ln(x+y) + \ln(x+y)^2 - \ln(x+y)^3 \\ = \ln(x+y) + 2 \cdot \ln(x+y) - 3 \cdot \ln(x+y) = 0 \end{aligned}$$

LINEARE GLEICHUNGEN

$$\textcircled{16.} \text{ a) } 2 \cdot (4 - 3x) = 3 \cdot (x + 2) - 16$$

$$\Leftrightarrow 8 - 6x = 3x + 6 - 16$$

$$\Leftrightarrow 18 = 9x$$

$$\Leftrightarrow 2 = x$$

$$\text{b) } 1,6x + 3,6 = 0,4x - 1,2$$

$$\Leftrightarrow 1,2x = -4,8$$

$$\Leftrightarrow x = -4$$

$$\text{c) } ax - a = bx - b$$

$$\Leftrightarrow ax - bx = a - b$$

$$\Leftrightarrow (a - b) \cdot x = a - b$$

$$\Leftrightarrow x = \frac{a - b}{a - b} = 1$$

$$\text{d) } t^2 x - 1 = x + t$$

$$\Leftrightarrow t^2 x - x = t + 1$$

$$\Leftrightarrow (t^2 - 1) \cdot x = t + 1$$

$$\Leftrightarrow x = \frac{t + 1}{t^2 - 1} = \frac{t + 1}{(t - 1) \cdot (t + 1)} = \frac{1}{t - 1}$$