Wurzelgleichungen

Bsp.

a) $\sqrt{x+4} - \sqrt{2x-1} = 0 \qquad | + \sqrt{2x-1}$

$\sqrt{x+4} = \sqrt{2x-1} \qquad | (\ldots)^2$

$x + 4 = 2x - 1 \qquad | -2x - 4$

$-x = -5 \qquad | \cdot (-1)$

$x = 5$

b) $\sqrt{2x+5} = (\underbrace{1}_{a} + \underbrace{\sqrt{x+2}}_{b})^2 \qquad | (\ldots)^2$

$2x + 5 = \underbrace{1^2}_{a^2} + 2 \cdot 1 \cdot \sqrt{x+2} + x + 2 \qquad | -1 - x - 2$

$a^2 + 2 \cdot a \cdot b + b^2$

$x + 2 = (2\sqrt{x+2})^2 \qquad | (\ldots)^2$

$x^2 + 2 \cdot 2 \cdot x + 2^2 = 2^2 \cdot (x+2)$

$x^2 + 4x + 4 = 4x + 8 \qquad | -4x - 8$

$x^2 - 4 = 0$

$x^2 + 0x - 4 = 0$

$x_{1,2} = -\frac{0}{2} \pm \sqrt{\left(\frac{0}{2}\right)^2 - (-4)}$

$= 0 \pm 2$

$x_1 = 2, \quad x_2 = -2$

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f) $2x = \sqrt{1-8x} - 5 \qquad | +5$

$\underbrace{2x}_{a} + \underbrace{5}_{b} = \sqrt{1-8x} \qquad | (\ )^2$

$\underbrace{(2x)^2}_{a^2} + \underbrace{2 \cdot 2x \cdot 5}_{2 \cdot a \cdot b} + \underbrace{5^2}_{b^2} = 1 - 8x$

$4x^2 + 20x + 25 = 1 - 8x \qquad | -1 + 8x$

$4x^2 + 28x + 24 = 0 \qquad | :4$

$x^2 + 7x + 6 = 0$

pq-Formel: $x_{1,2} = -\frac{7}{2} \pm \sqrt{\left(\frac{7}{2}\right)^2 - 6}$

$= -3{,}5 \pm 2{,}5$

$x_1 = -1 \,, \quad x_2 = -6$

h) $2\sqrt{x+1} - \sqrt{2x+3} = 1 \qquad | +\sqrt{2x+3}$

$2\sqrt{x+1} = 1 + \sqrt{2x+3} \qquad | (\ )^2$

$4 \cdot (x+1) = 1^2 + 2 \cdot 1 \cdot \sqrt{2x+3} + 2x+3 \qquad | -1 - 2x - 3$

$\cancel{2}x = \cancel{2} \cdot \sqrt{2x+3} \qquad | (\ )^2$

$x^2 = 2x + 3$

$x^2 - 2x - 3 = 0$

pq-Formel: $x_{1,2} = -\frac{-2}{2} \pm \sqrt{\left(\frac{-2}{2}\right)^2 - (-3)}$

$= 1 \pm 2$

$x_1 = 3 \,, \quad x_2 = -1$

Exponentialgleichungen

Bsp.

a) $3^{x+5} = 4^{x-2}$

$$\boxed{\ln(a^m) = m \cdot \ln(a)}$$

1. Mglk. (Logarithmusgesetze):

$$3^{x+5} = 4^{x-2} \qquad | \ln(\ldots)$$

$$\ln(3^{x+5}) = \ln(4^{x-2})$$

$$(x+5) \cdot \ln(3) = (x-2) \cdot \ln(4)$$

$$x \cdot \ln(3) + 5 \cdot \ln(3) = x \cdot \ln(4) - 2 \cdot \ln(4) \qquad | -x\ln(4) - 5\ln(3)$$

$$x \cdot \ln(3) - x \cdot \ln(4) = -2\ln(4) - 5\ln(3)$$

$$x \cdot (\ln(3) - \ln(4)) = -2\ln(4) - 5\ln(3) \qquad | : (\ln(3) - \ln(4))$$

$$x = \frac{-2\ln(4) - 5\ln(3)}{\ln(3) - \ln(4)} = 28{,}73\ldots$$

2. Mglk. (Potenzgesetze):

$$3^{x+5} = 4^{x-2}$$

$$3^x \cdot 3^5 = 4^x \cdot 4^{-2} \qquad | : 3^5, : 4^x$$

$$\frac{3^x}{4^x} = \frac{4^{-2}}{3^5}$$

$$\left(\frac{3}{4}\right)^x = 0{,}00026 \qquad | \ln(\ldots)$$

$$\ln\left(\left(\frac{3}{4}\right)^x\right) = \ln(0{,}00026)$$

$$x \cdot \ln\left(\frac{3}{4}\right) = \ln(0{,}00026)$$

$$x = \frac{\ln(0{,}00026)}{\ln(3/4)} = 28{,}73\ldots$$

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d) $e^{2+x} = 10 \qquad | \ln(\ldots)$

$$\ln(e^{2+x}) = \ln(10)$$

$$(2+x)\underbrace{\ln(e)}_{=1} = \ln(10)$$

$$2+x = \ln(10)$$

$$x = \ln(10) - 2$$

g) $5^x = 2 \cdot 3^x \qquad | \ln(\ldots)$

$$\ln(5^x) = \ln(2 \cdot 3^x) \qquad \ln(a \cdot b) = \ln(a) + \ln(b)$$

$$x \cdot \ln(5) = \ln(2) + \ln(3^x)$$

$$x \cdot \ln(5) = \ln(2) + x \cdot \ln(3) \qquad | -x \cdot \ln(3)$$

$$x \cdot \ln(5) - x \cdot \ln(3) = \ln(2)$$

$$x \cdot (\ln(5) - \ln(3)) = \ln(2)$$

$$x = \frac{\ln(2)}{\ln(5) - \ln(3)}$$

Logarithmusgleichungen

Bsp:

a) $\ln(x) - 2\ln(x+4) = 0$

$$\ln\left(\frac{x}{(x+4)^2}\right) = 0 \qquad \mid e^{(\dots)} \qquad\qquad e^{\ln x} = x$$

$$e^{\ln\left(\frac{x}{(x+4)^2}\right)} = e^0$$

$$\frac{x}{(x+4)^2} = 1 \qquad \mid \cdot (x+4)^2$$

$$x = (x+4)^2$$

$$x = x^2 + 8x + 16 \qquad \mid -x$$

$$0 = x^2 + 7x + 16$$

pq-Formel: $x_{1,2} = \dots$

b) $2\ln(x-1) + 3\ln(x+1) - 2\ln(x^2-1) = 0$

$$\ln\left(\frac{(x-1)^2\,(x+1)^3}{(x^2-1)^2}\right) = 0$$

$$\ln\left(\frac{\cancel{(x-1)}\,\cancel{(x-1)}\,\cancel{(x+1)}\cancel{(x+1)}(x+1)}{\cancel{(x^2-1)}\cancel{(x^2-1)}}\right) = 0$$

$$\ln(x+1) = 0 \qquad \mid e^{(\dots)}$$

$$x + 1 = e^0$$

$$x + 1 = 1 \qquad \mid -1$$

$$x = 0$$

c) $4\ln(\sqrt{x}) = 2 - \ln(x) \qquad | + \ln(x)$

$4\ln(\sqrt{x}) + \ln(x) = 2$

$\ln\left(\frac{(\sqrt{x})^4 x}{1}\right) = 2$

$\ln\left((x^{\frac{1}{2}})^4 \cdot x^1\right) = 2$

$\ln(x^3) = 2 \qquad | e^{(\ldots)}$

$x^3 = e^2 \qquad | \sqrt[3]{\ldots}$ bzw. $(\ldots)^{1/3}$

$x = \sqrt[3]{e^2} = 1{,}947\ldots$

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b) $\ln(x^2) + \ln(\sqrt{x}) = 10$

$\ln(x^2 \cdot \sqrt{x}) = 10$

$\ln(x^2 \cdot x^{1/2}) = 10$

$\ln(x^{2{,}5}) = 10 \qquad | e^{(\ldots)}$

$x^{2{,}5} = e^{10} \qquad | (\quad)^{\frac{1}{2{,}5}}$

$x = (e^{10})^{1/2{,}5}$

$x = e^4$

Lineare Gleichungssysteme

Bsp.

a) $2x + 4y = 10 \quad \cdot \frac{-3}{2}$ (Additionsverfahren)

$3x - 6y = -9$

$-3x - 6y = -15$ +

$3x - 6y = -9$

$0 - 12y = -24$

$3x - 6y = -9$

$y = 2$

$3x - 6 \cdot 2 = -9 \Rightarrow x = 1$

alternativ: Substitutionsverfahren

I $2x + 4y = 10$

II $3x - 6y = -9$

auflösen von I nach einer Variablen (z.B. x):

$2x + 4y = 10 \quad | -4y$

$2x = 10 - 4y \quad | :2$

$x = 5 - 2y \quad (*)$

einsetzen in II:

$$3 \cdot \underbrace{(5-2y)}_{x} - 6y = -9$$

$$15 - 6y - 6y = -9 \qquad | -15$$

$$-12y = -24 \qquad | : (-12)$$

$$y = 2$$

Einsetzen in (*):

$$x = 5 - 2 \cdot \underline{2}_{y}$$

$$x = 1$$