

A9

$$d) (-1)^{2n} = ((-1)^2)^n = 1^n = 1$$

$$e) \frac{(a^2 b^3 c^4)^3}{(a^3 b^2)^3 c^6} = \frac{(a^2)^3 (b^3)^3 (c^4)^3}{(a^3)^3 (b^2)^3 c^6} = \frac{a^6 b^9 c^{12}}{a^9 b^6 c^6}$$

$$= a^{6-9} b^{9-6} c^{12-6}$$

$$= a^{-3} b^3 c^6$$

$$h) \left(\left((z^{1/2})^{2/3} \right)^{3/4} \right)^{4/5} = \left((z^{1/2 \cdot 2/3})^{3/4} \right)^{4/5}$$

$$= (z^{1/2 \cdot 2/3 \cdot 3/4})^{4/5}$$

$$= z^{1/4 \cdot 2/3 \cdot 3/4 \cdot 4/5}$$

$$= z^{1/5}$$

Zusammenhang zwischen Wurzeln und Potenzen:

$$n\sqrt{x} = x^{1/n} \quad \text{bzw.} \quad \sqrt{x} = x^{1/2}$$

allgemeiner: $n\sqrt{x^m} = x^{m/n}$

Bsp.

Umschreiben von Potenzausdrücken in Wurzeln:

$$a) x^{3/2} y^{5/4} = \sqrt{x^3} \sqrt[4]{y^5}$$

$$b) \frac{x^{3/2} y^{7/3}}{x^{1/4}} = x^{3/2 - 1/4} y^{7/3} = x^{5/4} y^{7/3} = \sqrt[4]{x^5} \sqrt[3]{y^7}$$

$$c) x^{3/4} \cdot x^{-2} = x^{3/4 - 2} = x^{3/4 - 8/4} = x^{-5/4} = \sqrt[4]{x^{-5}}$$

$$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$$

A10

$$a) x^{-3/4} = \sqrt[4]{x^{-3}}$$

$$a^{-m} = \frac{1}{a^m}$$

$$x^{-3/4} = \frac{1}{x^{3/4}} = \frac{1}{\sqrt[4]{x^3}}$$

$$b) \left(\frac{1}{x}\right)^{-4/8} = \left(\frac{1}{x}\right)^{-1/2} = \frac{1}{\left(\frac{1}{x}\right)^{1/2}} = \frac{1}{\frac{1^{1/2}}{x^{1/2}}} = \frac{1}{\frac{1}{\sqrt{x}}}$$

$$= 1 \cdot \frac{\sqrt{x}}{1}$$

$$= \sqrt{x}$$

alternativ: $\left(\frac{1}{x}\right)^{-4/8} = \left(\frac{1}{x}\right)^{-1/2} = \sqrt{\left(\frac{1}{x}\right)^{-1}}$

$$= \sqrt{\frac{1}{\left(\frac{1}{x}\right)^1}}$$

$$= \sqrt{x}$$

$$c) a^{1/3} b^{2/3} = (a^1 b^2)^{1/3} = \sqrt[3]{a^1 b^2}$$

$$\text{bzw. } a^{1/3} b^{2/3} = \sqrt[3]{a} \sqrt[3]{b^2}$$

$$d) \left(\frac{x}{y}\right)^{1/2} \left(\frac{y^2}{x}\right)^{1/2} = \left(\frac{\cancel{x}}{y} \cdot \frac{y^{\cancel{2}}}{\cancel{x}}\right)^{1/2}$$

$$= y^{1/2} = \sqrt{y}$$

$$a^m \cdot b^m = (a \cdot b)^m$$

Bsp. Umformen von Wurzeln in Potenzen

$$a) \sqrt{x \sqrt{yx^2}} = \sqrt{x (yx^2)^{1/2}} = (x \cdot (yx^2)^{1/2})^{1/2}$$

$$= (x \cdot y^{1/2} x^{2 \cdot 1/2})^{1/2}$$

$$= (x^{1+1} y^{1/2})^{1/2}$$

$$= x^{2 \cdot 1/2} y^{1/2 \cdot 1/2}$$

$$= x \cdot y^{1/4}$$

$$b) \frac{\sqrt[3]{y^4 \sqrt{x^8}}}{\sqrt{yx^2}} = \frac{\sqrt[3]{y \cdot (x^8)^{1/4}}}{(yx^2)^{1/2}} = \frac{(y \cdot x^2)^{1/3}}{(yx^2)^{1/2}}$$

$$= \frac{y^{1/3} x^{2/3}}{y^{1/2} x^1}$$

$$= y^{1/3 - 1/2} x^{2/3 - 1}$$

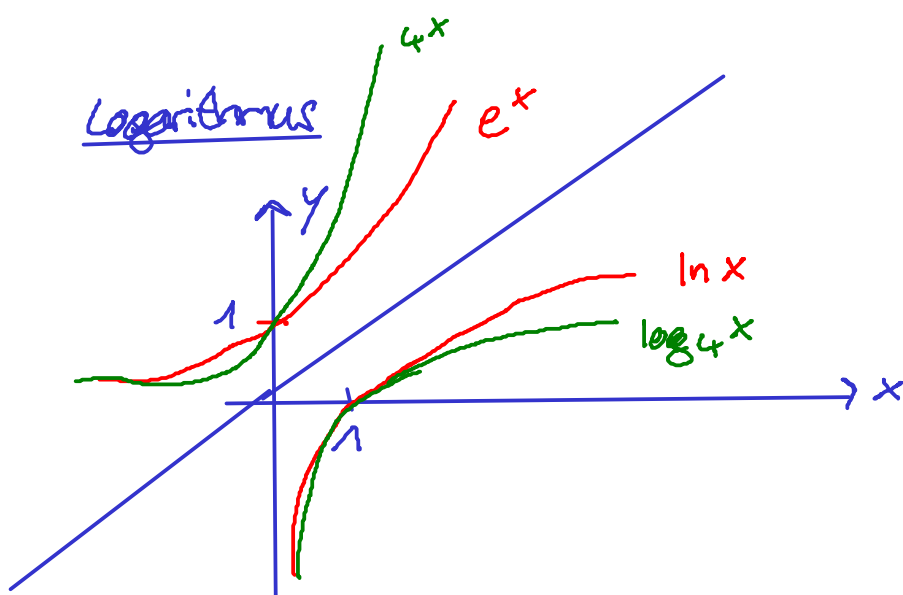
$$= y^{2/6 - 3/6} x^{2/3 - 3/3}$$

$$= y^{-1/6} x^{-1/3}$$

A12

$$\begin{aligned} \text{b) } \sqrt{x+4} \sqrt{2x+8} &= (x+4)^{1/2} (2x+8)^{1/2} \\ &= ((x+4)(2x+8))^{1/2} \\ &= ((x+4) \cdot 2 \cdot (x+4))^{1/2} \\ &= ((x+4)^2 \cdot 2)^{1/2} \\ &= ((x+4)^2)^{1/2} \cdot 2^{1/2} \\ &= (x+4) \cdot 2^{1/2} \end{aligned}$$

$$\begin{aligned} \text{a) } \frac{\sqrt[3]{(a+b)^4}}{\sqrt[3]{a+b}} &= \frac{((a+b)^4)^{1/3}}{(a+b)^{1/3}} = \frac{(a+b)^{4/3}}{(a+b)^{1/3}} = (a+b)^{4/3 - 1/3} \\ &= a+b \end{aligned}$$



$$e = 2,71\dots$$

Rechenregeln: $\ln(e^x) = x$ bzw. $e^{(\ln x)} = x$

$$\ln(1) = 0, \quad e^0 = 1$$

$$\ln(a) + \ln(b) = \ln(a \cdot b)$$

$$\ln(a) - \ln(b) = \ln\left(\frac{a}{b}\right)$$

$$\ln(a^m) = m \cdot \ln(a)$$

Alle oben genannten Regeln gelten für beliebige Basen
(d.h. man kann überall „ \ln “ durch „ \log_a “ ersetzen)

$$\left[\text{Formel für Basiswechsel: } \log_a(x) = \frac{\ln(x)}{\ln(a)} \right]$$

Bsp: Anwenden der Logarithmusgesetze

$$\begin{aligned} \text{a) } \ln((x+y)^2) + \ln((x+y)^3) &= \ln((x+y)^2 \cdot (x+y)^3) \\ &= \ln((x+y)^5) \\ &= 5 \cdot \ln(x+y) \end{aligned}$$

$$\text{b) } \frac{\ln(e^{x-y}) \cdot (x+y)}{x^2 - y^2} = \frac{(x-y)(x+y)}{x^2 - y^2} = \frac{x^2 - y^2}{x^2 - y^2} = 1$$

A14

$$\text{a) } \ln\left(\frac{x+y}{e}\right) = \ln(x+y) - \ln(e) = \ln(x+y) - \ln(e^1) = \ln(x+y) - 1$$

$$\text{b) } \ln\left(\frac{y}{z}\right) + \ln\left(\frac{z}{y}\right) = \ln\left(\frac{y}{z} \cdot \frac{z}{y}\right) = \ln(1) = 0$$

$$\begin{aligned} \text{c) } \frac{\ln(e^{x^2-y^2})}{x-y} - x &= \frac{x^2-y^2}{x-y} - x = \frac{(x-y)(x+y)}{x-y} - x \\ &= x+y - x = y \end{aligned}$$

Zusammenfassen mehrerer Logarithmen:

USP

$$a) 2 \ln x - 3 \ln y + 6 \ln(x-y) + 3 \ln(x+y)$$

$$\ln(a^m) = m \cdot \ln(a)$$

$$= \ln(x^2) - \ln(y^3) + \ln((x-y)^6) + \ln((x+y)^3)$$

$$= \ln\left(\frac{x^2 \cdot (x-y)^6 \cdot (x+y)^3}{y^3}\right)$$

$$b) 3 \ln(a) - 5 \ln(b) - 2 \ln(3b) + 8 \ln(a+b)$$

$$= \ln\left(\frac{a^3 (a+b)^8}{b^5 (3b)^2}\right)$$

AWS

$$e) \ln(x+y) + \ln((x+y)^2) - \ln((x+y)^3)$$

$$= \ln\left(\frac{(x+y)(x+y)^2}{(x+y)^3}\right)$$

$$= \ln\left(\frac{\cancel{(x+y)}^3}{\cancel{(x+y)}^3}\right)$$

$$= \ln(1)$$

$$= 0$$