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## 1. Rechengesetze und binomische Formeln

Bsp.

$$a) 3 + (-1) = 3 - 1 = 2$$

$$b) 3 + (-3xe^{2\sqrt{x}}) = 3 - 3xe^{\sqrt{x}}$$

$$c) 3 \cdot (7 - 2x) = 3 \cdot 7 - 3 \cdot 2x$$

$$d) (3 + 2y) \cdot (-7 + 8x) = 3 \cdot (-7) + 3 \cdot 8x + 2y \cdot (-7) + 2y \cdot 8x$$

$$= -3 \cdot 7 + 3 \cdot 8x - 2 \cdot y \cdot 7 + 2y \cdot 8x$$

allgemeine Regeln:

$$a \cdot (b + c) = a \cdot b + a \cdot c \quad (\text{Distributivgesetz})$$

$$a + (-b) = a - b$$

$$a - (-b) = a + b$$

Binomische Formeln:

$$(a + b)^2 [= (a + b) \cdot (a + b)] = a^2 + 2ab + b^2$$

$$(a - b)^2 [= (a - b) \cdot (a - b)] = a^2 - 2ab + b^2$$

$$(a+b) \cdot (a-b) = a^2 - b^2$$

Bsp.

$$a) \quad x^2 + 6x + 9 = \underbrace{x^2}_{a^2} + 2 \cdot \underbrace{3x}_{ba} + \underbrace{3^2}_{b^2} = (x+3)^2$$

$$b) \quad 3x^2 + 12x + 12 = 3 \cdot (x^2 + 4x + 4) \\ = 3 \cdot (x^2 + 2 \cdot 2x + 2^2) \\ = 3 \cdot (x+2)^2$$

$$c) \quad 3ab^2 - 9ab + 18a^2b = 3ab \cdot (b - 3 + 6a)$$

A4

$$c) \quad \underline{(x-1)(x+1)} \underline{(y-3)(y+3)} = \underline{(x^2-1^2)} \underline{(y^2-3^2)} \\ = x^2 \cdot y^2 - x^2 \cdot 3^2 - 1^2 y^2 + 1^2 3^2 \\ = x^2 y^2 - 9x^2 - y^2 + 9$$

A2

$$c) \quad (x-y)(x-1)(y-2) = (x^2 - x - xy + y) \cdot (y-2) \\ = x^2 y - \underline{xy} - xy^2 + y^2 - 2x^2 + 2x + \underline{2xy} \\ - 2y \\ = x^2 y - xy^2 + y^2 - 2x^2 + 2x + xy - 2y$$

A5

$$b) \quad 36u^2 - 12u + 1 = \underbrace{(6u)^2}_{a^2} - 2 \cdot \underbrace{6u}_{a} \cdot \underbrace{1}_{b} + \underbrace{1^2}_{b^2} = (6u - 1)^2$$

A4

$$a) \quad \left(\frac{1}{3} + x\right)\left(\frac{1}{3} - x\right) = \left(\frac{1}{3}\right)^2 - x^2 = \frac{1}{9} - x^2$$

## Bruchrechnung

### Rechenregeln

$$\frac{a}{b} \pm \frac{c}{d} = \frac{ad \pm bc}{b \cdot d}$$

$$\frac{1}{2} + \frac{3}{5} = \frac{1 \cdot 5 + 2 \cdot 3}{2 \cdot 5} = \frac{11}{10}$$

$$\frac{a}{b} \cdot \frac{c}{d} = \frac{a \cdot c}{b \cdot d}$$

$$\frac{1}{2} \cdot \frac{3}{5} = \frac{1 \cdot 3}{2 \cdot 5} = \frac{3}{10}$$

$$\frac{\frac{a}{b}}{\frac{c}{d}} = \frac{a}{b} \cdot \frac{d}{c} = \frac{a \cdot d}{b \cdot c}$$

$$\frac{\frac{1}{2}}{\frac{3}{5}} = \frac{1}{2} \cdot \frac{5}{3} = \frac{5}{6}$$

$$\frac{a}{b} = \frac{a \cdot c}{b \cdot c}$$

$$\frac{1}{2} = \frac{5}{10} = \frac{15}{30} \text{ usw.}$$

$$a = \frac{a}{1}$$

Spezialfälle:

$$\frac{-a}{b} = \frac{a}{-b} = -\frac{a}{b}$$

$$\frac{a}{\frac{c}{d}} = \frac{\frac{a}{1}}{\frac{c}{d}} = \frac{a}{1} \cdot \frac{d}{c} = \frac{ad}{c}$$

$$\frac{\frac{a}{b}}{c} = \frac{\frac{a}{b}}{\frac{c}{1}} = \frac{a}{b} \cdot \frac{1}{c} = \frac{a}{b \cdot c}$$

Bsp

$$\begin{aligned} \text{a) } \frac{3-5x}{x^2} + \frac{7}{x} &= \frac{3-5x}{x^2} + \frac{7 \cdot x}{x^2} = \frac{3-5x+7x}{x^2} \\ &= \frac{3+2x}{x^2} \end{aligned}$$

$$\begin{aligned} \text{b) } \frac{27a+3b}{3(a+b)} - \frac{5+10b}{a^2-b^2} &= \frac{\cancel{3} \cdot (9a+b)}{\cancel{3} \cdot (a+b)} - \frac{5+10b}{a^2-b^2} \\ &= \frac{(9a+b)(a-b)}{a^2-b^2} - \frac{5+10b}{a^2-b^2} \\ &= \frac{(9a+b)(a-b) - (5+10b)}{a^2-b^2} \\ &= \dots \end{aligned}$$

$$\begin{aligned} \text{c) } \frac{\frac{u+2v}{u}}{\frac{3u}{2u+v}} &= \frac{u+2v}{u} \cdot \frac{2u+v}{3u} = \frac{(u+2v)(2u+v)}{3u^2} \\ &= \frac{2u^2 + uv + 2uv + 2v^2}{3u^2} \\ &= \frac{2u^2 + 3uv + 2v^2}{3u^2} \end{aligned}$$

A7

$$\begin{aligned} \text{c) } 1 + \frac{1}{a-1} - \frac{1}{a+1} &= 1 + \frac{1 \cdot (a+1)}{(a-1)(a+1)} - \frac{1 \cdot (a-1)}{(a+1)(a-1)} \\ &= 1 + \frac{\cancel{a+1} - \cancel{(a-1)}}{a^2-1} \\ &= 1 + \frac{2}{a^2-1} \\ &= \frac{1 \cdot (a^2-1)}{1(a^2-1)} + \frac{2}{a^2-1} \\ &= \frac{a^2-1+2}{a^2-1} \\ &= \frac{a^2+1}{a^2-1} \end{aligned}$$

$$\begin{aligned} \text{b) } \frac{x^2-1}{x-1} + \frac{x^2+1-2x}{x-1} - 2x &= \frac{\cancel{x^2-1} + \cancel{x^2+1} - 2x}{x-1} - 2x \\ &= \frac{2x^2-2x}{x-1} - 2x \\ &= \frac{2x^2-2x}{x-1} - \frac{2x \cdot (x-1)}{x-1} \\ &= \frac{2x^2-2x-2x(x-1)}{x-1} \\ &= \frac{2x^2-2x-(2x^2-2x)}{x-1} \\ &= \frac{\cancel{2x^2} - \cancel{2x} - \cancel{2x^2} + \cancel{2x}}{x-1} \\ &= 0 \end{aligned}$$

A7

$$\begin{aligned} f) \frac{3}{cd} - \frac{5c}{cd+d^2} + \frac{7d}{c^2+cd} &= \frac{3}{cd} - \frac{5c}{d \cdot (c+d)} + \frac{7d}{c \cdot (c+d)} \\ &= \frac{3 \cdot (c+d)}{cd(c+d)} - \frac{5c \cdot c}{cd(c+d)} + \frac{7d \cdot d}{c \cdot d \cdot (c+d)} \\ &= \frac{3 \cdot (c+d) - 5c^2 + 7d^2}{cd(c+d)} \end{aligned}$$

## Potenzgesetze

$a^m$  ← Exponent  
↑  
Basis

$$a^m \cdot a^n = a^{m+n}$$

$$\frac{a^m}{a^n} = a^{m-n}$$

$$a^m \cdot b^m = (a \cdot b)^m$$

$$(a^m)^n = a^{(m \cdot n)}$$

Spezialfälle:

$$\frac{1}{a^m} = a^{-m} \quad \text{bzw.} \quad \frac{1}{a} = \frac{1}{a^1} = a^{-1}$$

Bsp.

$$a) \quad x^2 \cdot x^{1/4} = x^{2+1/4} = x^{9/4}$$

$$b) \frac{x^2 y^3 z^4}{x^{-4} z^2 y} = \frac{x^2}{x^{-4}} \cdot \frac{y^3}{y} \cdot \frac{z^4}{z^2} = x^{2-(-4)} \cdot y^{3-1} \cdot z^{4-2} \\ = x^6 y^2 z^2$$

$$c) \frac{(3^{x+y})^{x-y}}{3^{x^2}} = \frac{3^{(x+y)(x-y)}}{3^{x^2}} = \frac{3^{x^2-y^2}}{3^{x^2}} = 3^{\cancel{x^2}-y^2-\cancel{x^2}} \\ = 3^{-y^2}$$

$$d) (3xy^2z)^3 \cdot (9x^2yz)^{-1} = 3^3 x^3 (y^2)^3 z^3 \cdot 9^{-1} (x^2)^{-1} y^{-1} z^{-1} \\ = 3^3 x^3 y^6 z^3 \cdot 9^{-1} x^{-2} y^{-1} z^{-1} \\ = 27 x^{3-2} y^{6-1} z^{3-1} \frac{1}{9} \\ = 3 x y^5 z^2$$